No. of Printed Pages : 4

Sem-I-Phy-CC-I(R&B)

2023

Time - 3 hours

Full Marks - 60

*Answer **all groups** as per instructions.*
Figures in the right hand margin indicate marks.

GROUP - A

1. Answer <u>all</u> questions and fill in the blanks as required. [1 × 8

(a) The angle between $2\hat{i}+\hat{j}-\hat{k}$ and $\hat{i}-\hat{j}+\hat{k}$ is ________.

(b) Does the f(x) = | x | is continuous as well as differentiable at x = 0 ? (Yes / No / Cannot say)

(c) The second term in the binomial expansion of $(1 - x)^{-1}$ is ________

(d) The value of $f(x)\,\delta(x-a)$ is ________.

(e) If $\vec{\nabla}.\vec{A} = 0$, then $\vec{A}$ is called ________ vector.

(f) If u = y and v = x, then Jacobian $J\left(\frac{u, v}{x, y}\right)$ is ________.

P.T.O.

(g) The normal derivative of a scalar function is obtained by its ____________.

(h) The Stoke's theorem based on conversion of volume integral to surface integral. (True / False)

GROUP - B

2. Answer any eight of the following within two or three sentences each. [1½ × 8

(a) Evaluate curl $\phi\vec{A}$?

(b) Find the first order derivative of $f(x) = x^{\sin x}$.

(c) If $\vec{a}$ is a constant vector, then show that $\vec{\nabla}(\vec{r}\,.\,\vec{a}) = \vec{a}$.

(d) Sketch the function y = tan x without graph paper using scale on the axis.

(e) Evaluate $\vec{\nabla}(r^n)$.

(f) Prove that $\vec{\nabla}(\phi + \psi) = \vec{\nabla}\phi + \vec{\nabla}\psi$ where ϕ and ψ are scalar fields.

(g) Evaluate scale factors in circular cylindrical co-ordinate system.

(h) Find the unit normal vector to both $(\hat{i} - \hat{j} + \hat{k})$ and $(2\hat{i} + \hat{j} + \hat{k})$.

(i) Prove that $\delta(ax) = \frac{\delta(x)}{|a|}$.

(j) State Green's theorem.

GROUP - C

3. Answer <u>any eight</u> of the following within 75 words each. [2 × 8

(a) Define the Jacobian for transformation from Cartesian to spherical polar co-ordinates.

(b) Solve $2dx + \sec x \cos y \, dy = 0$ when $y(0) = 0$.

(c) Find the torque of force $\vec{F} = -3\hat{i} + \hat{j} + 5\hat{k}$ acting at (1, 3, –2) about origin.

(d) Obtain Taylor's series of $\cos x$ about origin.

(e) Does this equation $y\,dx - x\,dy = xy^3\,dy$ is exact ? If yes, solve it. If not, make it exact and solve.

(f) Prove that $\vec{\nabla}.\left(\vec{\nabla} \times \vec{A}\right) = 0$.

(g) Transform $\vec{A} = r\hat{e}_r + r\hat{e}_\theta$ from cylindrical co-ordinate to Cartesian co-ordinate.

(h) Find the equation of tangent plane to the surface $x^2 + y^2 - z^2 = 4$ at the given point (–1, 2, 1).

(i) Discuss about Wronskian.

(j) Express $\vec{\nabla}$ in spherical polar co-ordinate.

GROUP - D

*Answer **any four** questions within 500 words each.*

4. Discuss the properties of vector under rotation. [6

5. Solve $\sin^2 x \dfrac{d^2y}{dx^2} - 2y = 0$. [6

6. Find the equation of the tangent plane and normal line to the surface $x^2y + xz^2 = z - 1$ at the point $(1, -3, 2)$. [6

7. Find ∇^2 in spherical polar co-ordinate. [6

8. Establish the physical significance of divergence of a vector function. [6

9. The temperature at any point is given by scalar function $T = 400\,xyz^2$. Find the maximum temperature on the surface of unit sphere $x^2 + y^2 + z^2 = 1$. [6

10. Express velocity and acceleration in circular cylindrical co-ordinates. [6